STRAIN GAUGES ANALYSIS

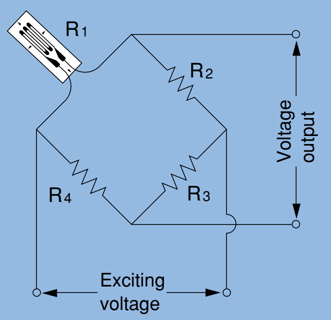
STRAIN GAUGE MEASUREMENT

When strain is generated in a test specimen and a strain gauge is attached, the strain is relayed via the gauge base(electrical insulation) to the resistance wire or foil in the gauge. As a result, the fine wire or foil experiences a variation in electrical resistance. This variation is exactly proportional to the strain.

$$\varepsilon = \frac{\Delta L}{L} = \frac{\Delta R / R}{K}$$

ε : strain measured
R : Gauge resistance
ΔR : Resistance change due to strain
K : Gauge Factor as shown on package

Normally, this resistance change is very small and requires a Wheatstone bridge circuit to convert it to voltage output.



The voltage output of a bridge circuit is given as follows.

$$e = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} E$$

e : Voltage output
E : Exciting voltage
R_1 : Gauge resistance
$R_2 \sim R_4$: Fixed resistance

Assuming the value R such that $R = R_1 = R_2 = R_3 = R_4$, the active gauge resistance varies to $R + \Delta R$ due to strain. Thus, the output voltage Δe(variation) due to the strain is given as follows.

$$\Delta e = \frac{\Delta R}{4R + 2\Delta R} E$$

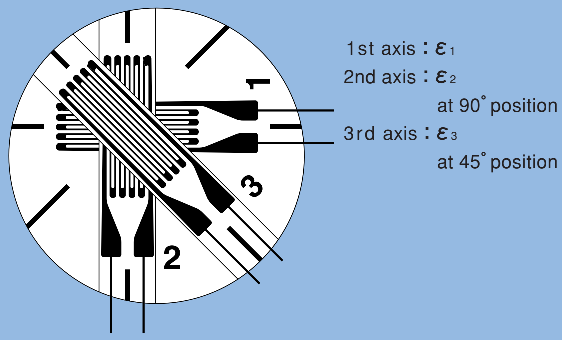
When $\Delta R \ll R$,

$$\Delta e = \frac{\Delta R}{4R} E = \frac{E}{4} K \varepsilon$$

The strain gauge is connected to a strainmeter, which provides the Wheatstone bridge circuit and exciting input voltage. The strain (ε) is measured on a digital or analog display.

CALCULATION FOR 3-ELEMENT ROSETTE ANALYSIS

The principal strain and its direction are calculated with a 45° /90° 3-element strain gauge, as described below.



Maximum principal strain

$$\varepsilon_{max} = \frac{1}{2}\left[\varepsilon_1 + \varepsilon_2 + \sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}\right]$$

Minimum principal strain

$$\varepsilon_{min} = \frac{1}{2}\left[\varepsilon_1 + \varepsilon_2 - \sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}\right]$$

Maximum shearing strain

$$\gamma_{max} = \sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}$$

Angle from ε_1 gauge to direction of principal strain

$$\phi_P = \frac{1}{2}\tan^{-1}\left\{\frac{2\varepsilon_3 - (\varepsilon_1 + \varepsilon_2)}{\varepsilon_1 - \varepsilon_2}\right\}$$

If $\varepsilon_1 > \varepsilon_2$, the angle to the maximum principal strain is rotated by Φ_P clockwise from the 1st axis, and the minimum principal strain is located at $\Phi_P + 90°$. If $\varepsilon_1 < \varepsilon_2$, the angle to the maximum principal strain is rotated by $\Phi_P + 90°$ clockwise from the 1st axis, and the minimum principal strain is located at Φ_P.

Maximum principal stress

$$\sigma_{max} = \frac{E}{1 - \nu^2}(\varepsilon_{max} + \nu\varepsilon_{min})$$
$$= \frac{E}{2}\left[\frac{\varepsilon_1 + \varepsilon_2}{1 - \nu} + \frac{1}{1 + \nu}\sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}\right]$$

Minimum principal stress

$$\sigma_{min} = \frac{E}{1 - \nu^2}(\varepsilon_{min} + \nu\varepsilon_{max})$$
$$= \frac{E}{2}\left[\frac{\varepsilon_1 + \varepsilon_2}{1 - \nu} - \frac{1}{1 + \nu}\sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}\right]$$

Maximum shearing stress

$$\tau_{max} = \frac{E}{2(1 + \nu)}\gamma_{max}$$
$$= \frac{E}{2(1 + \nu)}\sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}$$

,where E: Elastic modulus (Young's modulus)
ν : Poisson's ratio

NOTE

The above rosette analysis equations are based on the 3-element strain gauge shown in the diagram. When the order of the axis numbers is different or when the gauge is not a 90° rosette gauge, different equations must be used. Check the axis numbers of the applicable strain gauge before performing rosette analysis calculations.